**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2019**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 1 of term 4, 2019**

**Question 9 (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| With the data given, the interval beween data points is  hours  The trapezoidal rule yields that the integral is    Now  so that    Hence total electricity generated is approximately kWh | |
| Mathematical behaviours | Marks |
| * notes the interval between given data points * writes down the appropriate composite trapezoidal rule * computes the appropriate integral (-1 for one mistake) | 1  1  2 |

**Question 10(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  | | --- | --- | |  |  | | Vertical asymptote at | x-intercept at  (2,0) | | Horizontal asymptote at  y = 2 | Horizontal asymptote at | | |
| Mathematical behaviours | Marks |
| * states x-intercept correctly * states equation of horizontal asymptote correctly | 1  1 |

**Question 10(b)(i) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * indicates asymptotes, correctly * indicates (0,0) and correct behavior of curve for , ,, | 1  1 |

**Question 10(b)(ii) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * indicates asymptote x = 2, (0,0), (1,2) correctly * correct shape of the curve | 1  1 |

**Question 11(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Lines parallel to the x-axis below  cut the curve twice  is not a one-to-one function, therefore  does not exist | |
| Mathematical behaviours | Marks |
| * states the graph of does not pass the horizontal line test or states  is not a one to one function | 1 |

**Question 11(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| exists because | |
| Mathematical behaviours | Marks |
| * states and  correctly * states a reason | 1  1 |

**Question 11(c) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| =  , | |
| Mathematical behaviours | Marks |
| * forms a correct expression for | 1 |

**Question 11(d) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states that | 1 |

**Question 12(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| --------------(1)    --------------(2)      i.e.  i.e.  . | |
| Mathematical behaviours | Marks |
| * separates variables to form statement (1) * integrates correctly to form statement (2) or its equivalent * obtains expression for general solution | 1  1  1 |

**Question 12(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| i.e  or  is a circle with radius of 1 unit and centre at (0,0) | |
| Mathematical behaviours | Marks |
| * obtains correct expression for equation of circle * states radius and coordinates of centre correctly | 1  1 |

**Question 13(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| i.e | |
| Mathematical behaviours | Marks |
| * differentiates  implicitly correctly * differentiates  with respect to x correctly * obtains the required expression for | 1  1  1 |

**Question 13(b) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| ---------------------(1)      ------------------(2)  -------------------(3) | |
| Mathematical behaviours | Marks |
| * identifies  and uses  to form statement (1) * simplifies and separates variables to form statement (2) or its equivalent * anti-differentiates to obtain statement (3) or its equivalent * use to determine the constant of integration correctly * determines the velocity correctly when | 1  1  1  1  1 |

**Question 14(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  then  for  By de Moivre’s theorem then where  Hence the distinct roots are where ,  For arguments in the range given this is equivalent to | |
| Mathematical behaviours | Marks |
| * writes  in appropriate polar form * applies de Moivre’s theorem correctly * restricts the arguments of the solutions to the specified range | 1  1  1 |

**Question 14(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| We note that  Hence    From part (a) we conclude that    with the arguments as given in part (a) | |
| Mathematical behaviours | Marks |
| * realises that * divides equation through by thereby reducing the equation to the form in (a) * deduces the five roots of the modified equation | 1  1  1 |

**Question 15(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * obtains correct answer for * obtains correct answer for | 1  1 |

**Question 15(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| is normally distributed (\*) | |
| Mathematical behaviours | Marks |
| * uses normality * obtains correct answer | 1  1 |

**Question 15(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| So the required probability equals the area of the shaded triangle (\*)  i.e. 0.125 | |
| Mathematical behaviours | Marks |
| * obtains equality (\*) * obtains correct answer | 1  1 |

**Question 15(d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Because the sample size is large enough, the distribution of is approximately normal. (\*)  The mean is and the variance is (\*\*)  So | |
| Mathematical behaviours | Marks |
| * uses normal approximation * uses correct variance (\*\*) * obtains correct answer | 1  1  1 |

**Question 16(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| If then  and | |
| Mathematical behaviours | Marks |
| * uses the given change of variable to write integral in terms of * integrates correctly | 1  1 |

**Question 16(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  then  and so    Since  is a dummy variable this integral is equal to  as required | |
| Mathematical behaviours | Marks |
| * writes the integral in terms of the variable * realises that the minus sign in the derivative can be accounted for by interchanging limits | 1  1 |

**Question 16(c) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  then using the result of part (b) with gives that    Now and  so that    Thus using the result of part (a) | |
| Mathematical behaviours | Marks |
| * writes down the form of the integral using the structure of part (b) * establishes the forms and * simplifies the integrand * realises that the integral  is now part of the integral in part (a) and itself * combines the previous results to deduce the result | 1  1  1  1  1 |

**Question 17(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  then ,  and  Since  ,    Imaginary parts give    and real parts give | |
| Mathematical behaviours | Marks |
| * computes the values of and * substitutes into expression for and equates to zero * compares imaginary parts to deduce * compares real parts to deduce | 1  1  1  1 |

**Question 17(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| As has real coefficients, if  then also  Then  is a factor of  By long division | |
| Mathematical behaviours | Marks |
| * realises that must also be a root of * deduces a quadratic factor of the quartic * deduces the other quadratic using long division | 1  1  1 |

**Question 18(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| i.e. (\*)  Solving for gives  So the sample size should be at least | |
| Mathematical behaviours | Marks |
| * obtains equation (\*) * obtains correct answer | 1  1 |

**Question 18(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Confidence interval is , where  i.e.  i.e. | |
| Mathematical behaviours | Marks |
| * uses correct formula (\*) * obtains correct centre of CI (implicitly at least) * obtains width of CI (implicitly at least) | 1  1  1 |

**Question 18(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The evidence provided by the sample suggests that there has been a change in TV viewing time, but it is hardly compelling, since the old mean lies inside the confidence interval. | |
| Mathematical behaviours | Marks |
| * obtains correct conclusion * provides a valid reason | 1  1 |

**Question 19(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * substitutes into equation to solve for correctly | 1 |

**Question 19(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| As  i.e. as the number of nesting pairs of black terns approaches 500, their rate of increase approaches zero | |
| Mathematical behaviours | Marks |
| * states tends to zero and gives interpretation correctly | 1 |

**Question 19(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| -------------------- (1)          i.e | |
| Mathematical behaviours | Marks |
| * integrates correctly to form statement (1) or its equivalent * use the condition  to determine the correct constant of integration * obtains the required expression for *t* | 1  1  1 |

**Question 19(d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| . | |
| Mathematical behaviours | Marks |
| * expresses  in terms of an exponential function * rearranges and obtains an expression for *N* correctly | 1  1 |

**Question 19(e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| When  then or  to the nearest integer | |
| Mathematical behaviours | Marks |
| * substitutes value of  in the appropriate equation * solves correctly for *N* to the nearest integer | 1  1 |

**Question 20(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The given curve cuts the -axis at  Thus | |
| Mathematical behaviours | Marks |
| * identifies the limits of the integration * deduces the correct value of the area | 1  1 |

**Question 20(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Now | |
| Mathematical behaviours | Marks |
| * integrates correctly * simplifies to derive the correct answer | 1  1 |

**Question 20(c) (6 marks)**

|  |  |
| --- | --- |
| Solution | |
| For each value of there are two values and  for which  Now required volume obtained by rotating area about the -axis is    Now and are the roots of  so  Then we deduce that .  Hence    and if then    Then, if so | |
| Mathematical behaviours | Marks |
| * notes that for each value of in range there are two corresponding * derives expressions for and in terms of * writes down the integral for in terms of * uses an appropriate substitution to evaluate the integral * equates with * deduces the required value for | 1  1  1  1  1  1 |

**Question 21(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| So (\*)  and so the speed is given by | |
| Mathematical behaviours | Marks |
| * differentiates correctly (\*) * obtains correct answer | 1  1 |

**Question 21(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| (\*)  and  (\*\*)  So the height at time is  The maximum of occurs when and is  So the maximum height is cm. | |
| Mathematical behaviours | Marks |
| * solves for (\*\*) * find value of which maximizes * obtains correct answer | 1  1  1 |

**Question 21(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| The flight paths intersect if for some values of and  implies i.e.  Now  So the paths intersect at the point with coordinates (10,8,16)  Since the particles are at the intersection point at different times, there is no collision. | |
| Mathematical behaviours | Marks |
| * uses (\*) * shows that the paths intersect * shows that there is no collision | 1  1  1 |

**Question 21(d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| (\*)  So | |
| Mathematical behaviours | Marks |
| * finds (\*) * evaluates | 1  1 |

**Question 21(e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Using a graph of as a function of it is clear that the minimum occurs at .    (This can be checked using a derivative, but this is not required:  when  So is indeed a critical point.) | |
| Mathematical behaviours | Marks |
| * shows a sketch graph or calculates a derivative * answers correctly | 1  1 |

**Question 21(f) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| and  So (\*)  Since the dot product is the vectors are perpendicular. | |
| Mathematical behaviours | Marks |
| * evaluates and * evaluates dot product (\*) * gives a valid reason | 1  1  1 |